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chance, nor are they on that account of less value. It is of little popular interest. Its discoveries are seldom sensational. But its dignity and importance cannot be over-estimated. Of American Astronomers, the names of Hill and Newcomb will go down through the ages: their researches will never lose their importance. And whoever adds to this science is contributing to a knowledge which shall endure forever.

Baron Fourier said of La Place:

"Your successors, gentlemen, will witness the accomplishment of the great phenomena whose laws he discovered. They will observe in the motions of the Moon the changes which he predicted and of which he alone was able to assign the cause. The continued observation of Jupiter's satellites will perpetuate the memory of the inventor of the laws which govern them in their courses. The great inequality of Jupiter and Saturn, running through their long periods, and giving to these bodies new situations will recall without ceasing one of his most astonishing discoveries. These are the titles of a true glory which nothing can extinguish. The spectacle of the heavens will be changed, but at those remote epochs the glory of the inventor will continue forever; the traces of his genius bear the seal of immortality."

Chicago University, February 9th, 1896.

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M., (Princeton); Ph. D., (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

[Continued from January Number.]

Proposition XXII. *If two straights AB, CD existing in the same plane stand perpendicular to a certain straight BD; but AC joining these perpendiculars makes with them internal acute angles (in hypothesis of acute angle): I say (Fig. 26) the terminated straights AC, BD have a common perpendicular, and indeed within the limits fixed by the designated points A and C.*

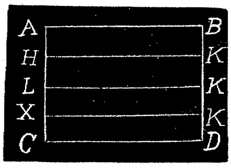

Fig. 26.

Proof. For if AB, CD are equal, it follows (from P. II) that the straight LK, by which these two AC and BD are bisected, will be to them a common perpendicular. But if either be the greater, as suppose AB; let fall to BD (according to Eu. I. 12) from any point L of AC the perpendicular LK, meeting the other *BD* in *K*. But it will meet it in some point *K* existing between the points *B* and *D;* otherwise (contrary to Eu. I. 17) the perpendicular *LK* would cut either *AB*, or *CD*, perpendicular to the same *BD*. So if the angles at the point *L* are not right, one of them will be acute and the other obtuse. Let the obtuse be toward the point *C*. But now *LK* is understood so to proceed toward *AB*, that it always stands at right angles to *BD*, and likewise opportunely increased, or diminished, in some point of it cuts the straight *AC*. It follows that the angles at the intersection points with *AC* cannot all be obtuse toward the parts of the point *C*, lest at length in that point *A*, where the straight *LK* is congruent with the straight *AB*, the angle at the point *A* toward the parts of the point *C* should be obtuse, when toward these parts it is by hypothesis acute. Since therefore the angle at the point *L* of this *LK* is by hypothesis obtuse toward the parts of the point *C*, the straight *LK* will not change over in this motion so as to make in some point of it with the straight *AC* an angle acute toward the parts of the aforesaid point *C*, unless, before, it changes over so as to make in some point of it with this *AC* an angle right towards the parts of this same point *C*. Therefore between the points *A*, and *L* will be some one intermediate point *H*, in which *HK* perpendicular to this *BD* is also perpendicular to to the other *AC*.

In a similar manner is shown to be present a certain *XK* between *LK*, *CD*, which is perpendicular both to the straight *BD*, and to the straight *AC*, if namely an angle at the point *L* is assumed to be obtuse toward the parts of the point A.

It follows therefore that the strights *AC*, *BD* will have a common perpendicular, and indeed within the limits fixed by the designated points *A*, and *C*, when the joins *AB*, *CD* exist in the same plane and are perpendicular to *BD*.

Quod erat, etc.

[To be Continued.]

INTRODUCTION TO SUBSTITUTION GROUPS.

By G. A. MILLER, Ph. D., Leipzig, Germany.

[Continued from January Number.]

(2) For each generating substitution s_α in a transitive group of degree m_1 find a substitution s_β which (*a*) interchanges the systems in the same way as s_α interchanges its elements, (*b*) has its k^{th} power in G_1 where k is the *order* of s_α, *i. e.* the lowest positive value of x which satisfies the equation $s_\alpha^x=1$, and (*c*) if s_β is